Reprojection for Patch-Based Denoising

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May 2010 - Statistical Models for Images
Luminy

Contents

Introduction and Framework

Framework

- Estimate an image $I^\star$ thanks to a noisy version I corrupted by an additive noise

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- Classical (Old) solution : averaging locally pixels values
- « Patches » approach : using pixels neighborhoods instead of using only pixels
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Images, Noise and Estimation

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- For a pixel (of interest) $i = (i_1, i_2) \in [\![1, N]\!]^2 = \Omega$
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- Quadratic Loss L_2, associated norm : $\| \cdot \|$

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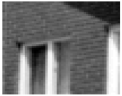

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- $I(i) = I^\star(i) + \sigma\mathcal{E}(i)$
- $\mathcal{E}$ i.i.d. standard Gaussian noise and known σ^2
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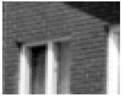

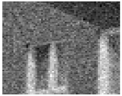

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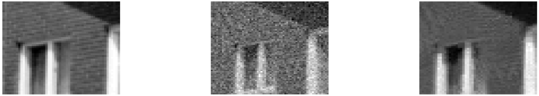

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Self - Similarity

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Denoising with Patches in Three Steps

Patches

- Patch P_i^I : Square Sub-Image "centered" on i, with width W :

$$P_i^I = \left(P_i^I[\delta]\right)_{\delta \in V_W} = \left(I(i+\delta)\right)_{\delta \in V_W}$$

with $\qquad V_W = \{-W_- \leq \delta_1, \delta_2 \leq W_+\}$

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Non Local Means (NL-Means)

Averaging Patches

- Estimate a patch (of interest) by averaging similar patches :

$$\widehat{P_i^I} = \sum_{k \in \Omega} \theta_{i,k} P_k^I$$

- Patchs useless if $\theta_{i,k}$ does not depend on P_k^I and P_i^I !

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Average in a "small" searching zone Ω_R (if enough time take $\Omega_R = \Omega$) :

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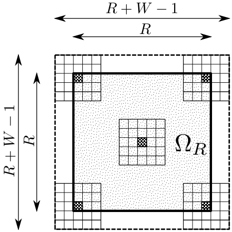

NL-Means in 3 parameters : Patch (W), Searching (R), Bandwidth (h)

Easy to implement and to explain : few parameters !

NL-Means [BCM05]

- Use weights defined by $\theta_{i,k} = \frac{K_h(\|P_i^I - P_k^I\|)}{\sum_{k'} K_h(\|P_i^I - P_{k'}^I\|)}$
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Common parameters choices for 512×512 images

- Patch width : $1 \leq W \leq 16$ (W increases with σ)
- Searching zone width : $10 \leq R \leq \cdots$ ($R = 21$ [BCM05])
- Kernel bandwidth h : harder to choose, increases with σ, often $h \propto \sigma$

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The Larger R the longer the algorithm

- Time limit : Algorithmic Complexity $O(N^2R^2)$, leading to choose a small searching zone Ω_R

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- Performance limit : too many small weights damage the procedure...

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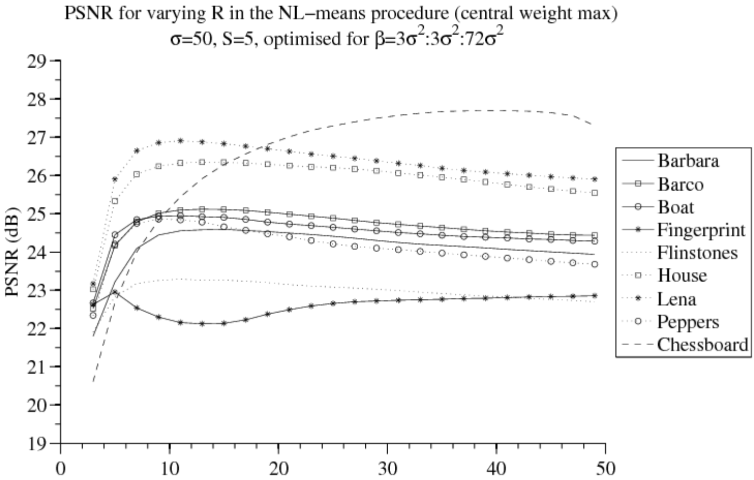
Influence of R



$\sigma = 50$, $W = 5$, oracle h (optimized)

Kernel choice K

Usual Suspects $\cdots$

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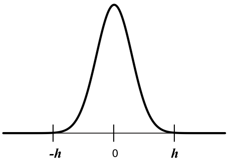

(a) Gaussian Kernel

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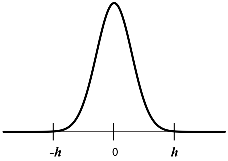

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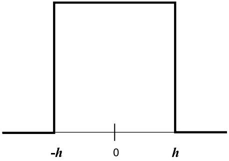

(b) Flat Kernel

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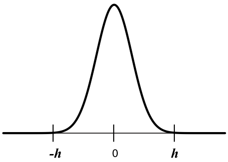

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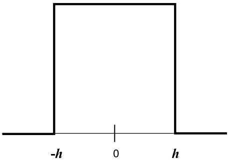

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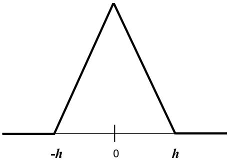

(c) Triangular Kernel

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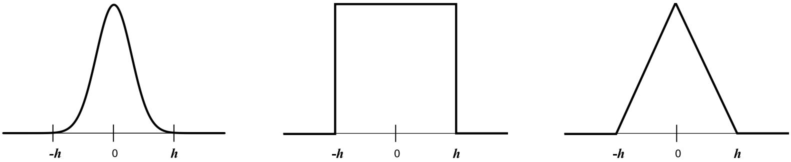

(a) Gaussian Kernel (b) Flat Kernel (c) Triangular Kernel

Previous slide : rather use a **compact** kernel since small weights degrade the performance

Patch of Interest's weight(s)

Comparing a patch with itself : "Central Weight Problem"

- Maximum weight $\theta_{i,j}$ reached for $i = j$, with K symmetric and non-increasing on $\mathbb{R}^+$
- Overestimate the patch of interest weight

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Central weight (before normalisation)

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- Unbiased Risk Estimate Weights [Sal10] :

$$\mathbb{E}(\|P_j^I - P_i^I\|^2 - 2\sigma^2 W^2) = \|P_j^{I^\star} - P_i^{I^\star}\|^2$$

  With Gaussian kernel : Substitute $\theta_{i,i} = K_h(2W^2\sigma^2)$ to $K_h(0)$, others unchanged
- Normal Weights : no weight changed
- Zero Weights : $\theta_{i,i} = 0$ (standard measurement)

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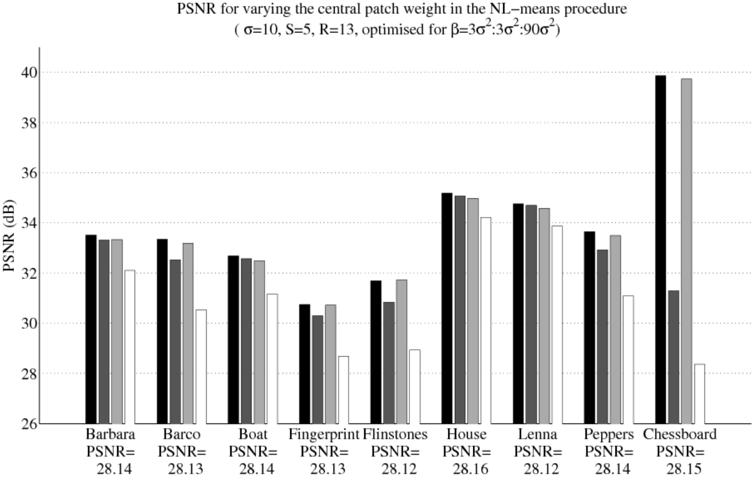
Central Weight and PSNR



$\sigma = 10$

Flat Kernel Advantages

Previous slide : use a kernel constant around 0 to avoid the Central Weight problem

Keep or Kill the patches

- $K_h(x) = \mathbb{1}_{[-1,1]}(\frac{x}{h})$
- Same weights for all selected patches

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Selecting the bandwidth h with hypothesis testing

- H_0 :"$P_i^{I^\star} = P_j^{I^\star}$" vs H_1 :"$P_i^{I^\star} \neq P_j^{I^\star}$"
- Under H_0, $\|P_i^I - P_j^I\|^2/(2\sigma^2) \sim \chi^2(W^2)$
- Choose $h^2 = 2\sigma^2 q_\alpha^{W^2}$ for a confidence level α (e.g $\alpha = 99\%$)

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Seminal Reprojection I : Central

Central Reprojection

- Define a center for each patch : $P_i^I[0]$

(a) Odd width (b) Even width

- Use the same weights measuring the similarity of patches for the similarity of the centers :

$$\widehat{I}(i) = \sum_{k \in \Omega_R} \theta_{i,k} P_k^I[0] = \widehat{P_i^I}[0]$$

Central Reprojection in Practice

(a) Original

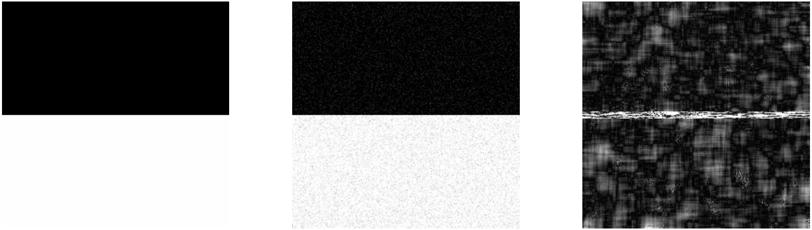
(b) Noisy PSNR $= 22.07$

(c) PSNR $= 27.60$

FIG.: (a) Original image, (b) Noisy image with $\sigma = 20$, (c) Denoised image (**Flat kernel being used until the end**)

Parameters : $R = 9$, $W = 9$, $h^2 = 2\sigma^2 q_{0.99}^{W^2}$.

Edges Artifacts on a Toy Example



(a) Original (b) Noisy PSNR $= 22.07$ (c) PSNR $= 45.78$

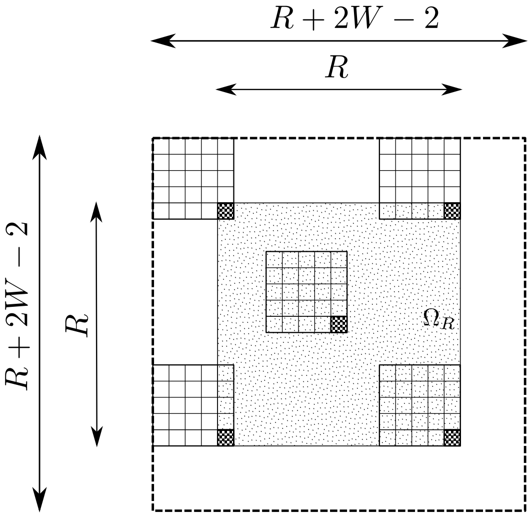
FIG.: (a) Original image, (b) Noisy image with $\sigma = 20$, (c) Absolute difference between the denoised image and the original image (the whiter the worse)

Parameters : $R = 21,\ W = 9, h^2 = 2\sigma^2 q_{0.99}^{W^2}$.

Edges Artifacts on Natural Images

Searching Zone, Influence Zone and Patches

Patches width $W = 5$, searching zone width $R = 15$



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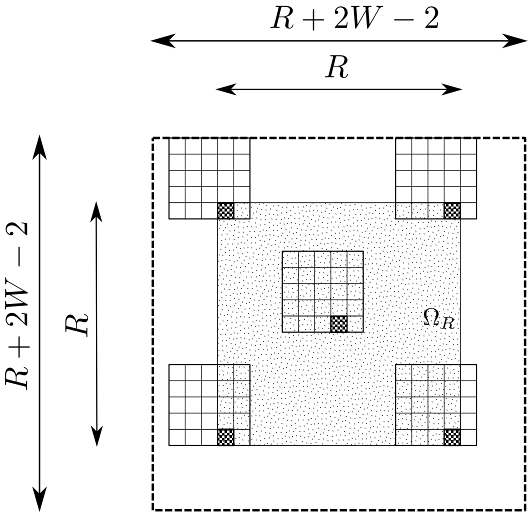

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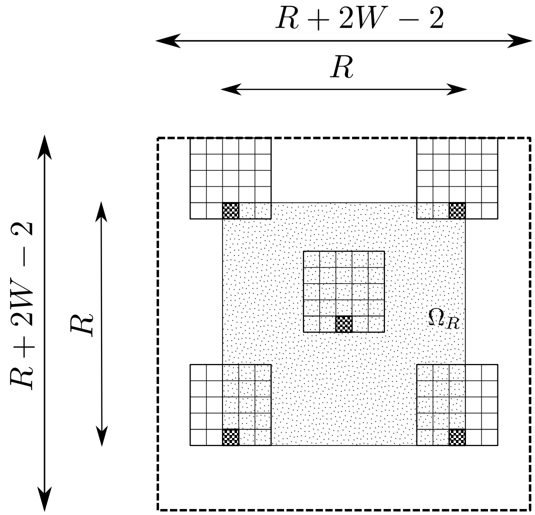

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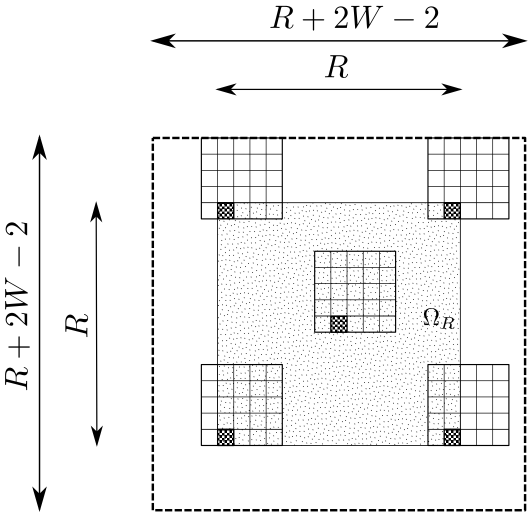

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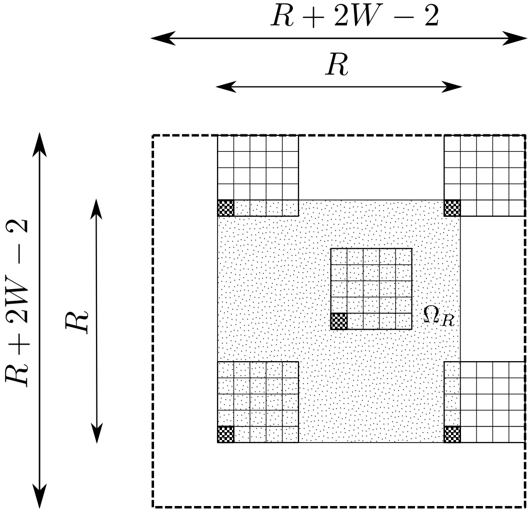

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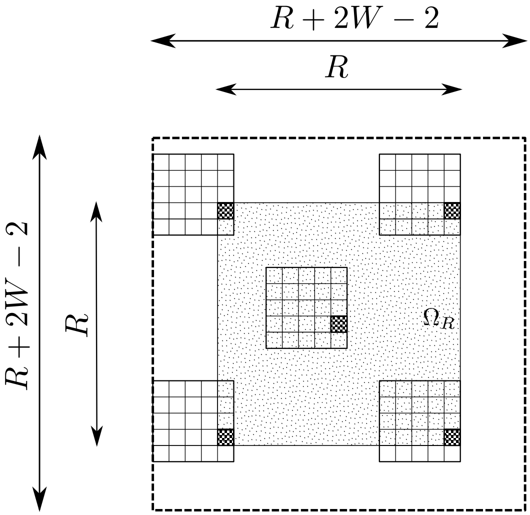

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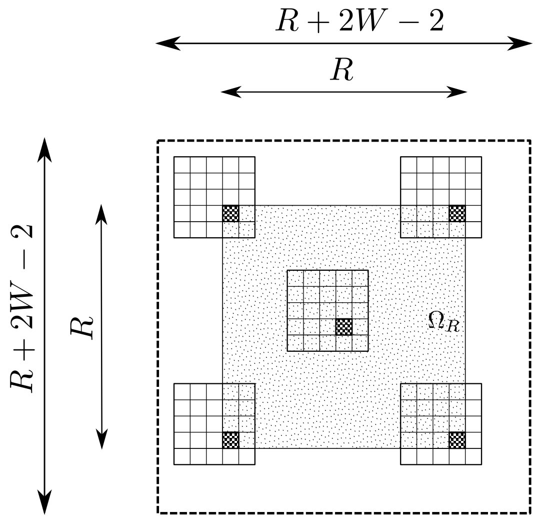

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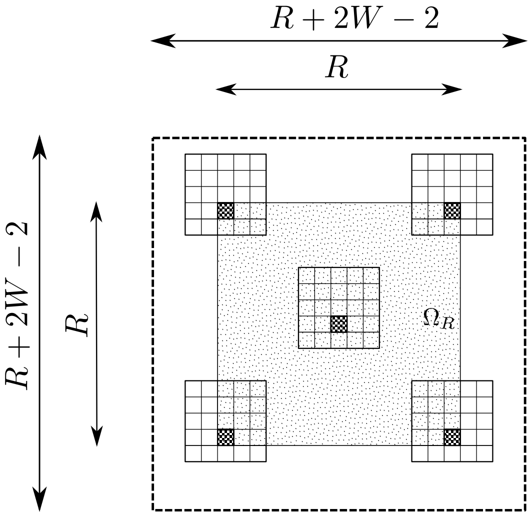

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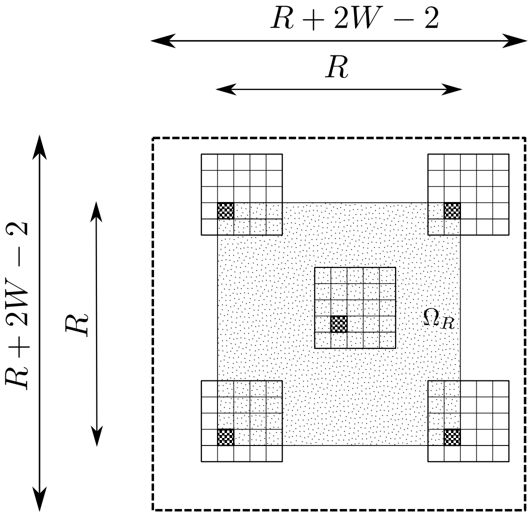

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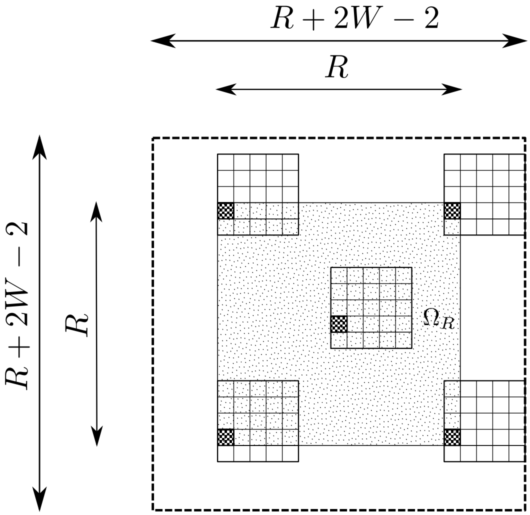

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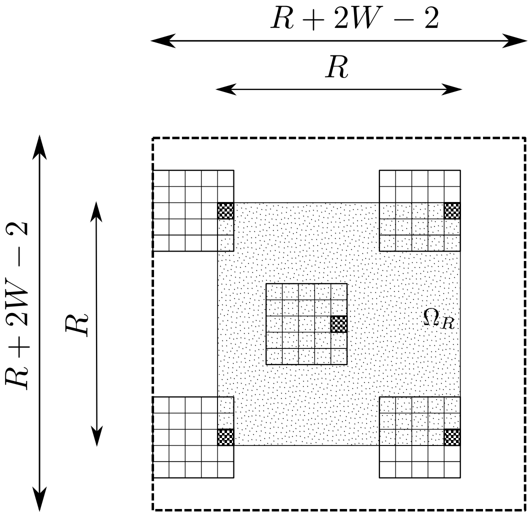

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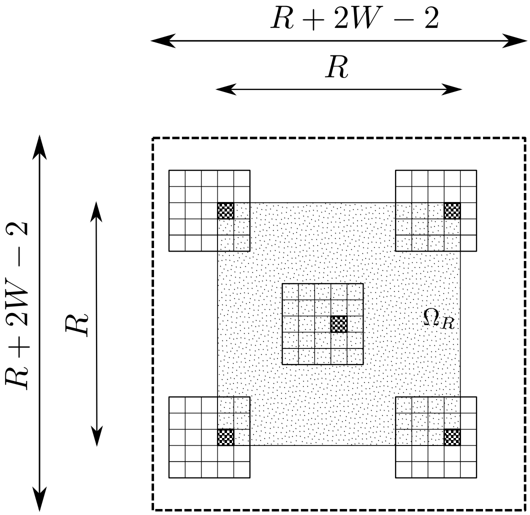

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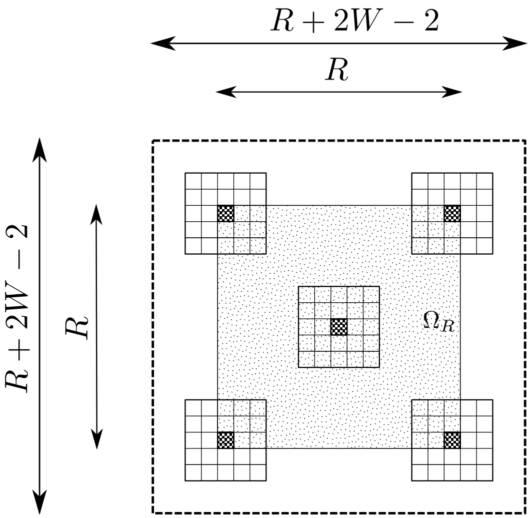

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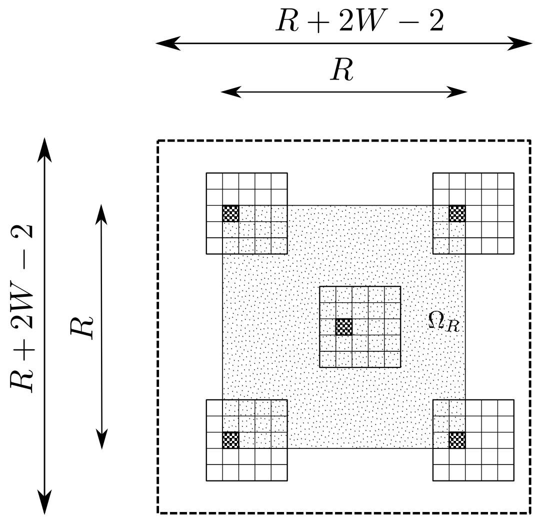

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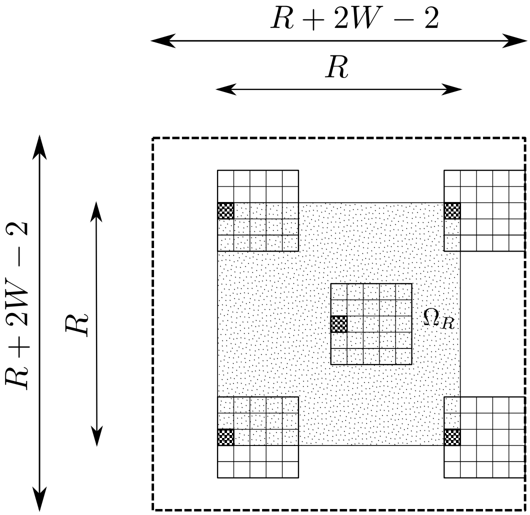

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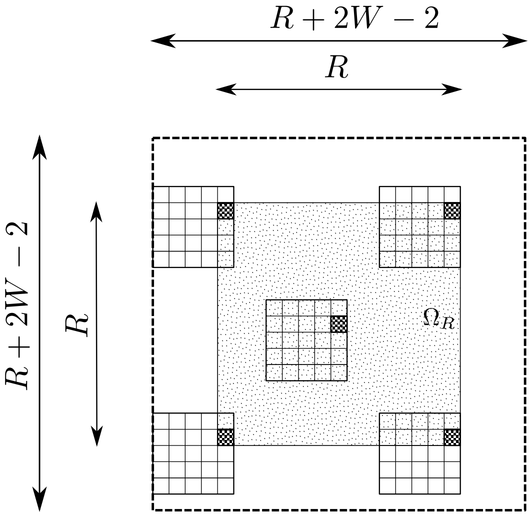

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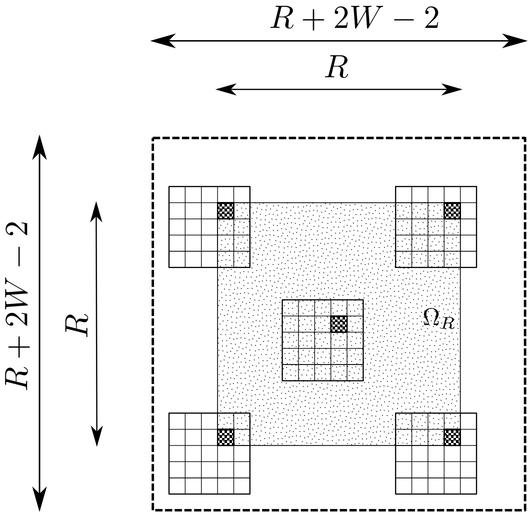

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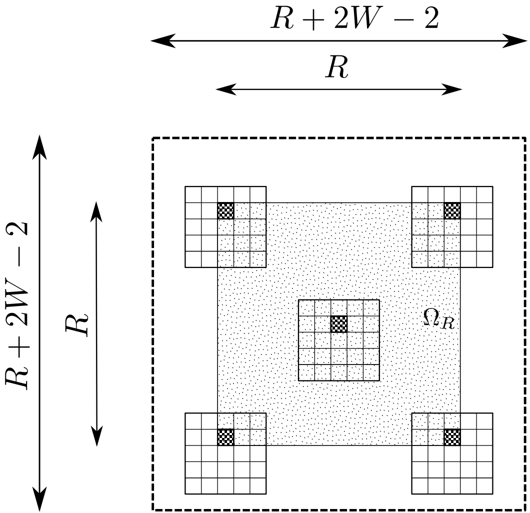

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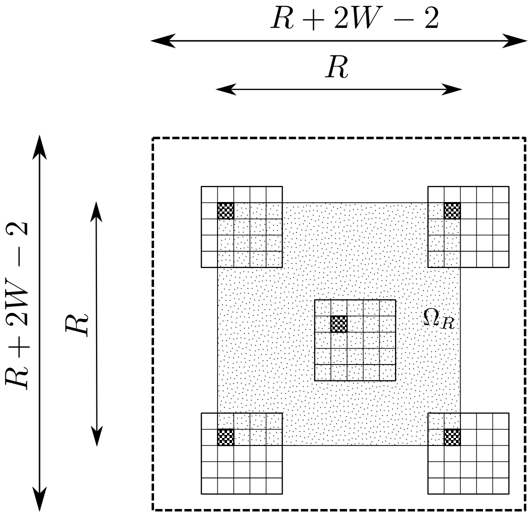

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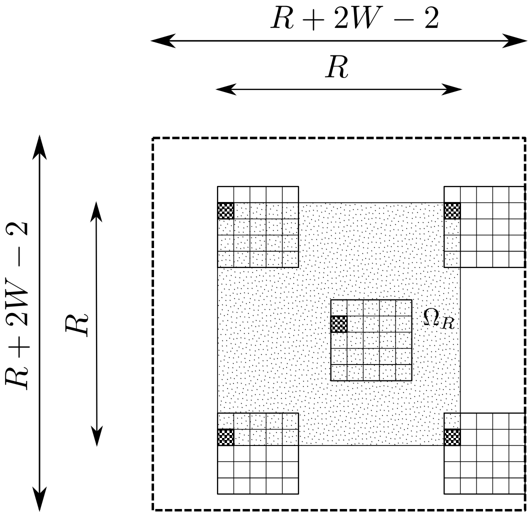

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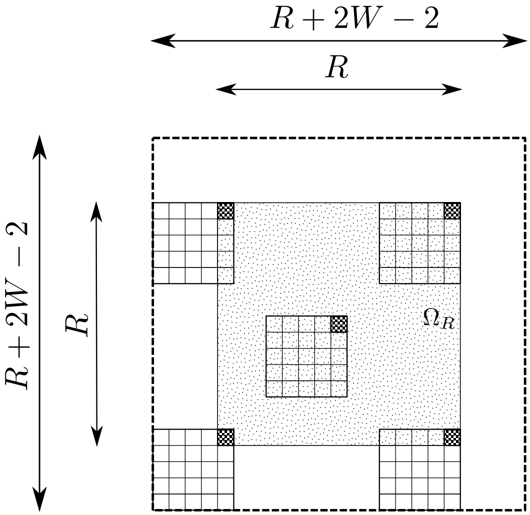

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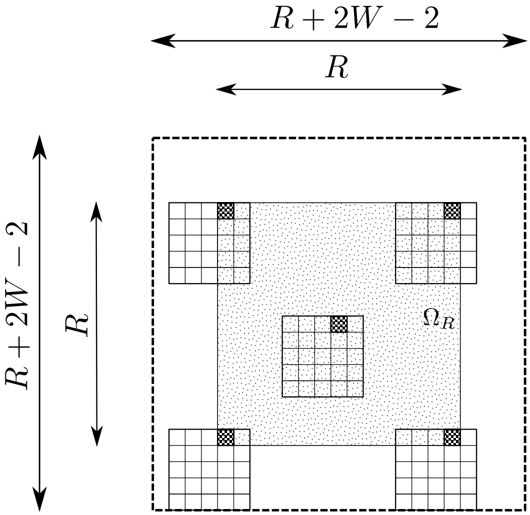

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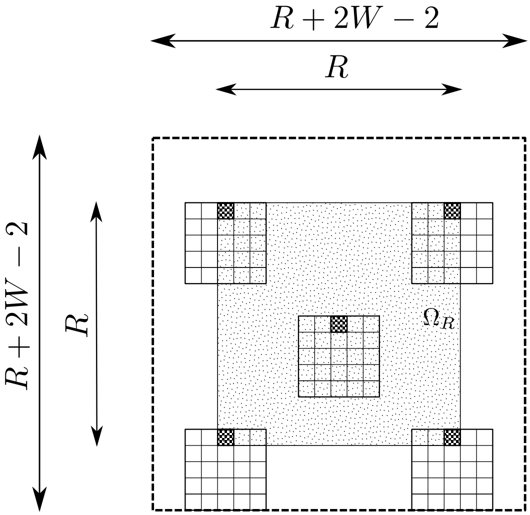

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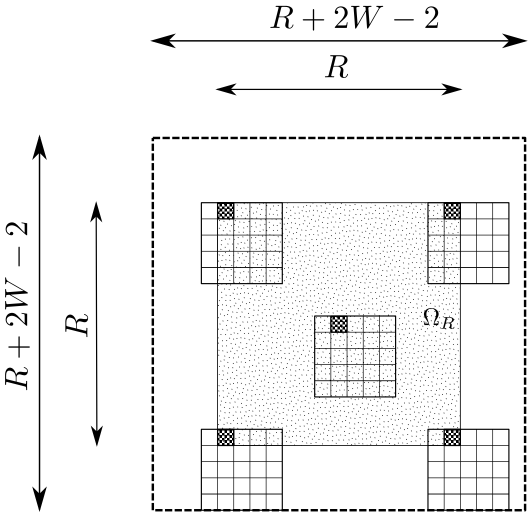

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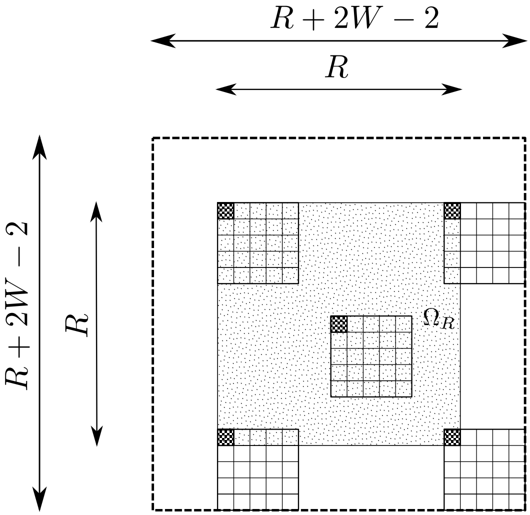


Seminal Reprojection II : Uniform Average of Estimators (Uae)

Uae Reprojection [BCM05]

- Each pixel belongs to W^2 patches $\Rightarrow$ each pixel has W^2 estimators :

$$\forall \delta \in V_W, I(i) = P^I_{i-\delta}[\delta]$$

- Average those estimators with uniform weights

$$\widehat{I}_{\mathrm{Uae}}(i) = \frac{1}{|V_W|} \sum_{\delta \in V_W} \widehat{P^I_{i-\delta}}[\delta]$$

- In practice : important PSNR improvement

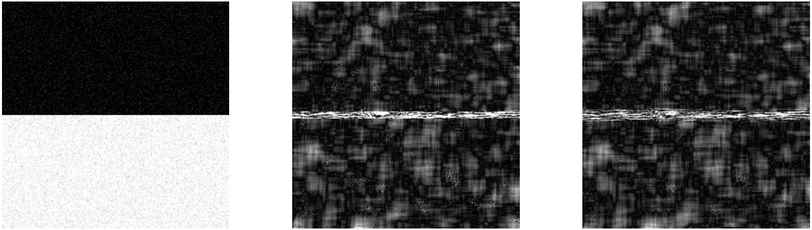
Uae : Natural Images

(a) Noisy (b) Central PSNR $= 27.60$ (c) Uae PSNR $= 28.65$

FIG.: (a) Noisy image ($\sigma = 20$), (b) Central Reprojection, (c) Uae Reprojection.

Parameters : $R = 9,\ W = 9, h^2 = 2\sigma^2 q_{0.99}^{W^2}$.

Uae : Edges Artifacts on a Toy Image



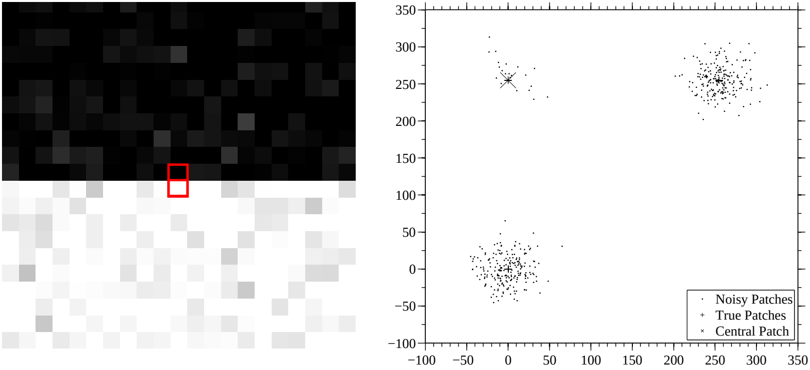
(a) Noisy (b) Central PSNR $= 45.78$ (c) Uae PSNR $= 46,51$

FIG.: (a) Noisy image ($\sigma = 20$), (b) Central Reprojection, (c) Uae Reprojection.

Parameters : $R = 21,\ W = 9, h^2 = 2\sigma^2 q_{0.99}^{W^2}$.

Uae : Edges Artifacts on Natural Images

Artifacts Explanation



(a) Toy Image Close Up (b) Clusters of Patches

FIG.: (a) Searching zone width $R = 21$, $\sigma = 20$, (b) Patches distribution in the searching zone (a). Patches are vertical with size 2×1

Minimizing Variance Reprojection

Naive Idea : assume the bias ≈ 0, choose the estimate with minimum variance

Minimum Variance

$$\widehat{I}_{\mathrm{Min}}(i) = \widehat{P^I_{i-\hat{\delta}}}[\hat{\delta}]$$

$$\text{with} \qquad \hat{\delta} = \arg\min_{\delta \in V_M} \mathrm{Var}\left(\widehat{P^I_{i-\delta}}[\delta]\right)$$

- Rough approximation of the variance [KB06] :

$$\mathrm{Var}\left(\widehat{P^I_{i-\delta}}[\delta]\right) \approx \frac{\sum_k \theta^2_{i-\delta,k}}{(\sum_k \theta_{i-\delta,k})^2}\sigma^2$$

- Flat Kernel case : Choose the estimator selecting the maximum number of patches (candidates)

$$\mathrm{Var}\left(\widehat{P^I_{i-\delta}}[\delta]\right) \approx \frac{\sigma^2}{N_{i-\delta}}$$

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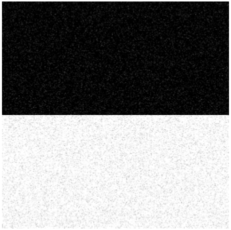
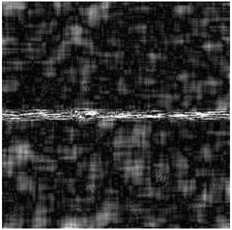
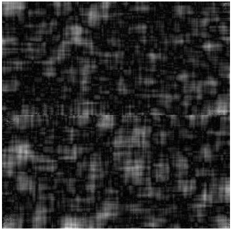
Minimizing Variance Reprojection in Practice



(a) Noisy



(b) Uae :PSNR $= 46, 51$



(c) Min : PSNR $= 48.10$

FIG.: (a) Noisy image ($\sigma = 20$), (b) Uae Reprojection , (c) Min Reprojection.

Parameters : $R = 21,\ W = 9, h^2 = 2\sigma^2 q_{0.99}^{W^2}$.

Minimizing Variance Reprojection in Practice

(a) Noisy (b) Uae : $\underline{\text{PSNR} = 28.65}$ (c) Min : $\underline{\text{PSNR} = 27.09}$

FIG.: (a) Noisy image ($\sigma = 20$), (b) Uae Reprojection , (c) Min Reprojection.

Parameters : $R = 9,\ W = 9, h^2 = 2\sigma^2 q_{0.99}^{W^2}$.

Crenelated Edges

Weighted Averaged Reprojection (Wav)

Naive Idea II : assume bias ≈ 0, and remember that an average of unbiased is unbiased. Thus take the weighted average minimizing the variance :

Wav Reprojection

$$\widehat{I}_{\mathrm{Wav}}(i) = \sum_{\delta \in V_W} \beta^\star_\delta \widehat{P^I_{i-\delta}}[\delta]$$

$$\text{where} \quad (\beta^\star_\delta)_{\delta \in V_W} = \underset{\beta \in \mathbb{R}^{W^2}}{\arg\min} \operatorname{Var}\left(\sum_{\delta \in V_W} \beta_\delta \widehat{P^I_{i-\delta}}[\delta] \right)$$

s.t. $$\sum_{\delta \in V_W} \beta_\delta = 1$$

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s.t.

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Weighted Averaged Reprojection

Lagrangian Optimization

$$\beta_\delta^\star = \left[\mathrm{Var}\left(\widehat{P_{i-\delta}^I}[\delta]\right)\right]^{-1} / \sum_{\delta \in V_W} \left[\mathrm{Var}\left(\widehat{P_{i-\delta}^I}[\delta]\right)\right]^{-1}$$

- Rough approximation of the variance :
$$\mathrm{Var}\left(\widehat{P_{i-\delta}^I}[\delta]\right) \approx \frac{\sum_{\Omega_R} \theta_{i,k}^2}{\left(\sum_{\Omega_R} \theta_{i,k}\right)^2} \sigma^2$$
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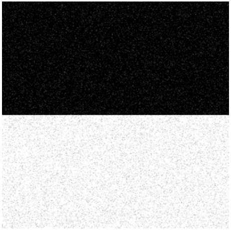
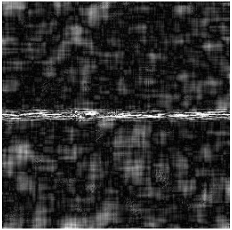
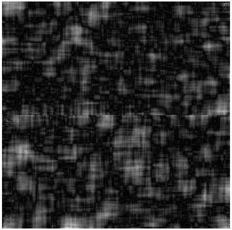
Weighted Averaged Reprojection in Practice



(a) Noisy



(b) Uae :PSNR $= 46, 51$



(c) Min : PSNR $= 47, 77$

(a) Noisy

(b) Uae : PSNR $= 28.65$

(c) Wav : PSNR $= 29.08$

FIG.: (a) Noisy image ($\sigma = 20$), (b) Uae Reprojection , (c) Wav Reprojection.
Parameters : $R = 21, 9$ (up, down), $W = 9, h^2 = 2\sigma^2 q_{0.99}^{W^2}$.

Uae

Wav

Conclusion

Improvements with Wav Reprojection

- Numerical (PSNR)
- Visual : edges better preserved

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Work in progress...

- Other reprojections
- Last obvious drawback : small region with no redundancy, patch size should change according to local redundancy
- Theoretical results (cf. Erwan's Talk)

Conclusion

Improvements with Wav Reprojection

- Numerical (PSNR)
- Visual : edges better preserved

Work in progress...

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- Theoretical results (cf. Erwan's Talk)

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